TODAY YOU WILL...

> LEARN ABOUT ENLARGEMENTS AND SCALE FACTORS IN TWO DIMENSIONS.

STAYING ON THE PAGE

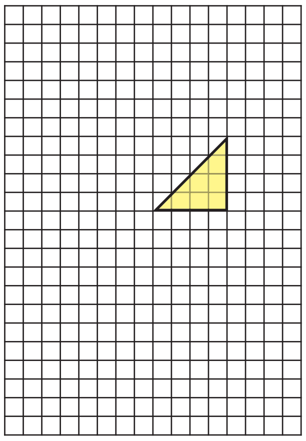
IN THIS LESSON, A PROBLEM OF SPACE AVAILABILITY IS TURNED INTO AN OPPORTUNITY FOR EXPLORING HOW ENLARGEMENTS WORK, SAYS COLIN FOSTER...

Students can often learn to perform a procedure successfully without much understanding of what they are doing – let alone why they are doing it! A good way to help them get a deeper understanding is to present them with an *inverse task*, where they have to try to do the opposite process. This is usually harder and, unless they have learned it as a standard procedure, requires some careful thinking to unpick what they know. In the topic of enlargements of 2D shapes, students are normally asked to enlarge a given shape with a given scale factor about a given centre of enlargement, or to describe in these terms an enlargement that has already been drawn. Sometimes they are asked to find the centre of enlargement or scale factor for a given enlargement. In this lesson, students are asked to decide where the centre of enlargement can go for a given shape with a scale factor of 3 enlargement so that no part of the image will go off the edge of the grid. Solving this problem involves students in intelligent trial and error as they experiment with various possible centres of enlargement in order to locate the locus of possible positions.

STARTER ACTIVITY

Q ***Look at this triangle drawn on a grid (p.57). If I said that I wanted you to make an enlargement of it, what information would I have to give you so that I got exactly the enlargement that I was thinking of?*** Students may say things like 'How much bigger', and you could ask them whether they can remember the formal term for this ***(scale factor)***.

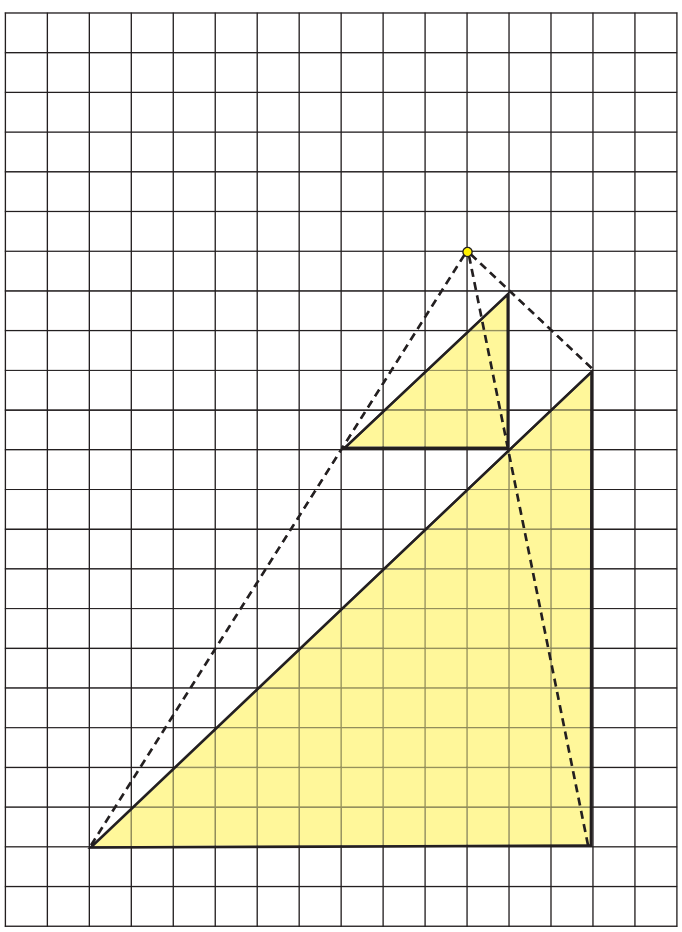
NB – YOU CAN PRINT OUT A FULL-SIZED EXAMPLE OF THIS WORKSHEET FROM: http://www.foster77.co.uk/Staying%20on%20the%20Page%20WORKSHEET.pdf



Q ***I want the scale factor to be 3, please. What does that mean?***
Students might say that it means that the new shape will be '3 times as big' as the old shape. This is right in terms of lengths, but in fact the area will be 9 times as big, so '3 times as big' by itself is a bit ambiguous.

Q ***Anything else?***
Students sometimes forget that to determine the location of the image the ***centre of enlargement*** must also be specified.

Q ***What does 'centre of enlargement' tell you?***
This is easier to explain with an example, so draw in a dot exactly where it is shown in the diagram below, and let the students go through the procedure for carrying out an enlargement with a scale factor of 3. They may need reminding how to do it!
All measurements begin at the centre of enlargement (the dot in the diagram). The distance from this dot to each vertex must be multiplied by 3 and the new distance measured ***from the dot.*** (A common mistake is to measure the new distance ***from the vertex***, resulting in a scale factor 4 enlargement instead!)



Q ***How can we check that we have done the enlargement correctly?***
- The sides of the shape should be 3 times as long in the image as they are in the object.
- The angles at each vertex should be the same (45°, 45°, 90°).
- Each new side should be parallel to the old side.
- The new shape should look the same, but bigger, and it should be in the same orientation.

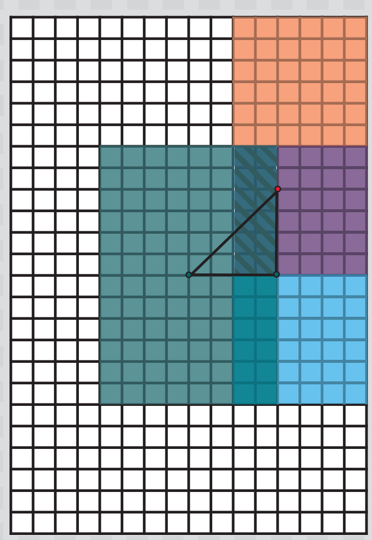
In exams, another way to check is that no part of your image should go off the edge of the grid provided! That is going to be an important idea in the main activity.

MAIN ACTIVITIES

Q ***I chose a position for the centre of enlargement so that the image would lie completely on the grid. No part of the image goes off the edge of the grid. Do you think that will always happen? [No.] Can you suggest a position for the centre of enlargement which will mean that part of the image does go off the edge of the grid when we do a scale factor 3 enlargement?***
A student could suggest a point, and you could begin the construction to see what happens when part of the triangle goes off the edge of the grid. It is easy to find positions for the centre of enlargement where this happens.

Q ***I want you to work out where the centre of enlargement can be so that when you do a scale factor 3 enlargement no part of the image goes off the grid. I would like you to find all the possible positions.***
Students might ask whether we are considering only positions that are at grid points or whether we are including in-between positions as well. You could say that for now we will just consider grid points, but later students could look at other points too. Students might also ask whether it is OK if a vertex lies ***exactly*** on the edge of the grid, and you could say that that is fine.

This is a demanding task. Students will need to experiment with different positions, and this will involve them in lots of useful practice of the procedure. At the same time, they will be



HOME LEARNING

Students could try moving the original triangle to a new position, nearer one of the edges, and see what effect this has on the permissible region. They could produce a new diagram to show what they find.

anticipating each time what is going to happen and thinking about the effects of moving the centre of enlargement around.

One way to think about this problem is to consider each vertex separately. For each vertex, find the locus of points where the centre of

INFORMATION CORNER

ABOUT OUR EXPERT

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ADDITIONAL RESOURCES

A GOOD WEBSITE FOR EXPLORING THE EFFECTS OF CHANGING THE POSITION OF THE CENTRE OF ENLARGEMENT FOR DIFFERENT SHAPES IS WWW.WALDOMATHS.COM/ENLARGE1L.JSP. A NICE GAME BASED ON THIS IS AT HTTP://AUTOGRAPH-MATHS.COM/ACTIVITIES/MRBARTON/LOADING.HTML?ACTIVITY=GOLFENLARGE.HTML.

STRETCH THEM FURTHER

EXPLORING WHAT HAPPENS WITH A SHAPE WITH MORE SIDES, SUCH AS A RECTANGLE, WOULD PRESENT FURTHER CHALLENGES. WOULD THE PERMISSIBLE REGION STILL BE A RECTANGLE, OR WOULD IT BE SOME OTHER SHAPE?

enlargement can be such that the image of that vertex lies on the grid. The areas in which this happens for each of the three vertices are shaded in different colours in the diagram on the left. (Note that the green rectangle is larger than the other two, both of which get cropped by the right-hand edge of the grid.) Where these three rectangles overlap is shaded with slanted lines. A centre of enlargement at any point in this slanted region (including the boundary) will lead to all three vertices lying on the grid, which means that the whole of the triangle must lie on the grid.

SUMMARY

You could conclude the lesson with a plenary in which learners talk about what they have found out. Some students may have just one feasible position for the centre of enlargement, whereas others might have found a collection of points. Can anyone prove that they have found all the possible points? One way to do this is by thinking about each vertex in isolation, as in the diagram on the previous page.